Addition

Stage 1: Physical Objects

Use physical objects for addition. Children count both set of objects and then count

If I have 3 apples and 2 apples, I have 5 apples altogether.

Move from physical objects to pictorial recording, but not too soon

If I have 3 apples and 2 apples, I have 5 apples altogether.

3 2

3 + 2 = 5

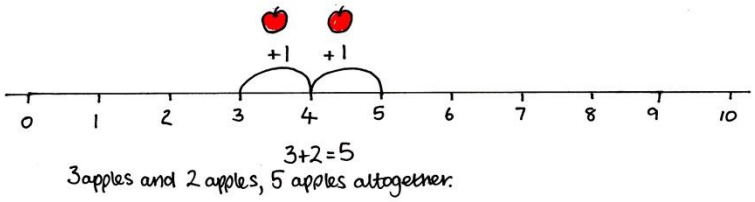
Stage 2: Using a bead string

Add 2 numbers by counting on

Stage 3: Structured number line

Link the practical activity of counting on to recording using a structured number line

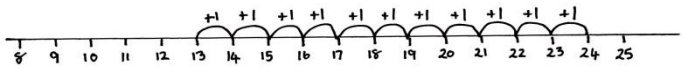
If I have 3 apples and 2 apples, I have 5 apples altogether

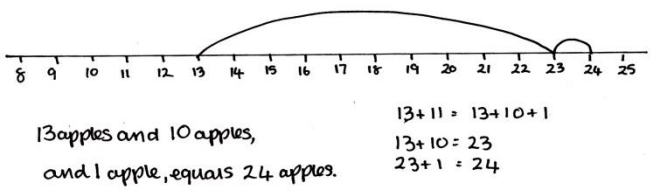


NB: Use of a structured number line must be embedded before moving on to unstructured/empty number line

Stage 4: The structured number line

Children start by using steps of 1. Children will then refine this by jumping in steps of 10 other numbers based on their increasing mental use of number facts.



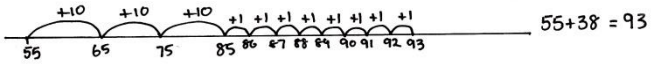


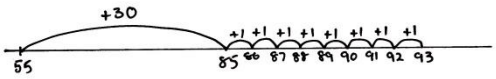
Stage 5: The empty number line

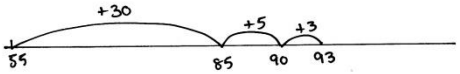
The empty number line helps to record the steps on the way to calculating the total. It is an informal jotting that should be used to support mental calculation strategies.

Counting on in multiples of 100, 10, 1

There are 38 fiction books and 55 non-fiction books in the library. How many books are there in the library?







Stage 6: Partitioning

The next stage is to record mental methods using partitioning.

47 + 76 = 40 + 70 = 110
7 + 6 = 13
110 + 13 = 123

Note that the children's recording of this method may be more informal (jottings) and they may only record the steps towards the final answer e.g.
36 + 48 = 70 + 14 = 84
Partitioned numbers are then written under one another. This mirrors the column method and also links mental methods.

47 40 + 7
+76 70 + 6
110 +13 =123

Stage 7: Expanded method in columns

Adding the most significant digit first
(as this compliments mental strategies)

47
+ 76
110 (70+40)
13 (7+6)
123

Adding the ones first
(once children have gained confidence)

47
+ 76
13 (7+6)
110 (70+40)
123

Stage 8: Column method

Exchanging is recorded below the line.

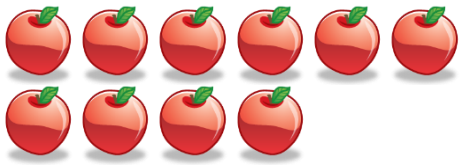
47 258 366
+ 76 + 87 + 458
123 345 824
1 1 1 1 1 1

Subtraction

Stage 1: Physical Objects

Use physical objects for subtraction. Children count the objects and then take away the number of objects to be subtracted

If I have 6 apples and I eat 2, I have 4 apples left.



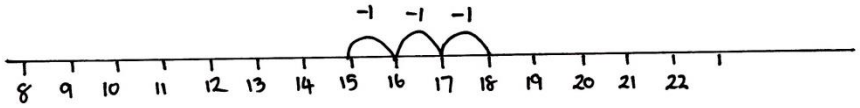
Move from physical objects to pictorial recording, but not too soon

If I have 6 apples and I eat 2, I have 4 apples left.

6-2=4

Stage 2: Structured number line

If a farmer has 18 apples on a tree and he picks 3. How many apples will be left?



18 apples, take away 3 apples, equals 15 apples.

18 - 3 = 15

Stage 3: Unstructured number line

15 - 7

15 – 7 = 8

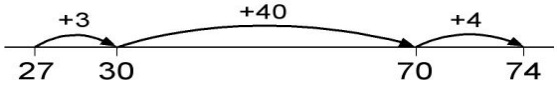
74 - 27

74 – 27 = 47

The steps may be recorded in a different order:

Children must have a secure understanding of the link between counting back and subtraction before using a counting up method.

Difference: 74 – 27



Stage 4: Expanded layout, leading to column method

This method would be supported with practical apparatus. Once confident this then leads to the column method. The amount of time spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

Example: **563 − 241**

Expanded method *leading to*

500 + 60 + 3	563
- 200 + 40 + 1	- 241
300 + 20 + 2	322

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

Example: **74 − 27**

	60 14	60 14
70 + 4	~~7~~0 + ~~4~~	74
- 20 + 7	- 20 + 7	- 27
	40 + 7	47

For the column method (decomposition) start from the right-hand column. In this method 2 minus 7 cannot be done without using negative numbers (do not swap the digits over!). So exchange a ten for ten ones. This leaves 2 tens and 12 ones. In effect, what has been done is to partition the 32 into 20 + 12, which is the same as in the expanded method but more efficiently recorded.

Column method

932 – 457 becomes

$$\begin{array}{rccc} & 8 & 12 & 1 \\ & \cancel{9} & \cancel{3} & 2 \\ - & 4 & 5 & 7 \\ \hline & 4 & 7 & 5 \\ \hline \end{array}$$

Multiplication

Early multiplication is about counting repeated groups the same size. Moving towards recording might be labelling sets using digit cards, or drawing a picture. Early division is about sharing objects into equal groups and counting how many are in each group
"Grouping" and "sharing" are different aspects of division:

I have 12 apples to share between 3 people. How many will each person have?
I have 12 apples to pack. Each pack holds four apples. How many packs do I need?

Stage 1: Physical objects

Use physical objects to complete multiplication as repeated addition
If I have six teddy bears, how many legs will there be?

Move from physical objects to pictorial recording, but not too soon.
If I have six teddy bears, how many legs will there be?

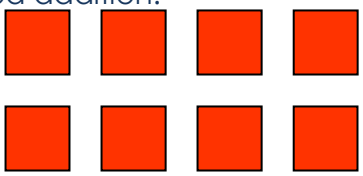
2 + 2 + 2 + 2 + 2 + 2 = 12 legs

2 x 6 = 12

Stage 2: Arrays

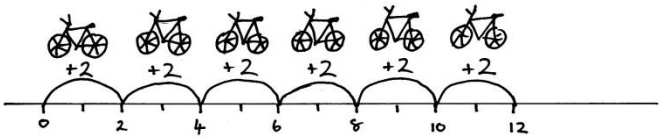
Arrays are important because they provide a good visual image of the calculation tha
closely to the concept of repeated addition.

4 x 2 = 8 2 x 4 = 8



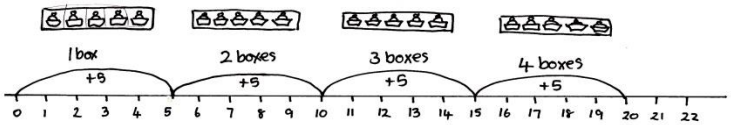
Stage 3: Number line

I have 6 bicycles…how many wheels altogether?



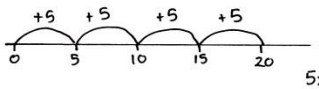
2 + 2 + 2 + 2 + 2 + 2 = 12 wheels.

There are 5 cakes in one box. How many cakes in 4 boxes?



5 x 4 = 20

5 cakes in each box. 4 boxes equals 20 cakes altogether.



5 x 4 = 20

Stage 4: Grid method

This method links to mental methods. Children are encouraged to begin with an estimation.

38 × 7 =

	7
30	210
8	56

210 + 56 = 266

Children will need to use their tables and place value knowledge e.g.
'I know 3 x 7 = 21 so 30 x 7 = 210'

The grid method may be the main method used by some children throughout KS2 and can be used with larger numbers and decimals.

Stage 5: Expanded short multiplication

The next step is to represent the method of recording in a column format, but showing the working. *leading to:*

38 x 7 =

30 + 8
7 x

56 (7 x 8)
210 (30 x 7)

266

t	u
3	8
	7
5	6
2	1 0
2	**6 6**

Stage 6: Short multiplication

The exchanged digits are recorded below the line.

38
× 7
266
5

Children need to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage e.g. this calculation involves adding 210 and 50 mentally.

Stage 7: Long multiplication (multiplying by two-digits)

Children should be encouraged to estimate first.
124 × 26 is approximately 120 × 30 = 3600 (using known fact 12 x 3 = 36)

124 × 26 becomes

	1	2	
	1	**2**	**4**
×		**2**	**6**
2	**4**	**8**	**0**
	7	**4**	**4**
3	**2**	**2**	**4**
1	1		

Division

Early division is about sharing objects into equal groups and counting how many are in each group
"Grouping" and "sharing" are different aspects of division:

I have 8 apples to share between 2 people. How many will each person have?



This is a 'sharing' task. Sharing the apples between 2 by the 'one for you one for me' approach until apples are gone. How many does each person have?

I have 8 apples to pack. Each pack holds two apples. How many packs do I need?

This is a 'grouping' task as you are putting the apples into groups of 2 and counting the 'groups'.

Stage 1: Division as sharing

Start with division as sharing, using concrete objects:

If I have 12 lollipops and I can share them among 4 people, how many lollipops will each person have?

Stage 2: Division as grouping

Numbers to 50 ÷ U

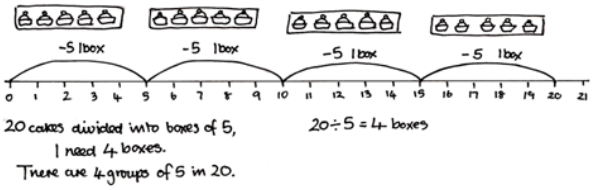
Represent repeated subtraction (grouping) as division use practical and informal written methods and related vocabulary to division, including calculations with remainders

If I have 12 lollipops and want to put 4 lollipops in each bag, how many bags will I need?

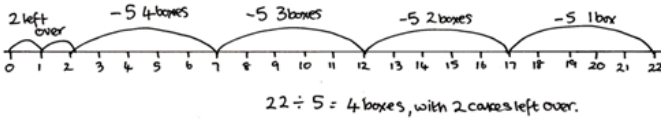
12 ÷ 4 = 3

Stage 3: Number line

I have 20 cakes, I can fit 5 cakes in a box.
How many boxes will I need?



What if I had 22 cakes to pack?



Stage 4: Chunking method (using multiples and known facts)

This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks.

For 81 ÷ 3

```
   27
3 ⟌81
- 30      10 x 3
  51
- 30      10 x 3
  21
- 21      7 x 3
   0
```

Ans. 27

For 72 ÷ 5

```
   14   remainder 2
5 ⟌72
- 50      10 x 5
  22
- 20      4 x 5
   2
```

Ans. 14 r.2

For the calculation 81 ÷ 3 the divisor is 3. So children subtract multiples of 3.
This method allows children to be successful with limited times tables knowledge (x10, x5, x2) although this will require more steps, 'chunks.'
As children gain confidence they can refine this method and reduce recording by using the largest possible multiples. Children need to be confident with multiplication facts and understanding of place value.

For 196 ÷ 6

```
   32 remainder 4
6 ⟌196
- 180     30 x 6
   16
-  12      2 x 6
    4
```

Stage 6: Short division

Short division can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.

98 ÷ 7 becomes

```
   1  4
7 | 9  8
    2
```

Answer: 14

432 ÷ 5 becomes

```
   8  6  r 2
5 | 4  3  2
    3
```

Answer: 86 remainder 2

Stage 7: Long division (when dividing by a 2 digit number)

Long division involves subtracting the largest possible multiple. Children need quick recall of multiplication tables and related division facts, which they then apply when working with larger numbers.

432 ÷ 15 becomes

```
      2  8  r 12
1  5 | 4  3  2
       3  0  0
       1  3  2
       1  2  0
          1  2
```